## Fractal dimensions of a green broccoli and a white cauliflower

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## Abstract

The fractal structures of a green broccoli and a white cauliflower are investigated by box-counting method of their cross-sections. The capacity dimensions of the cross-sections are  $1.78 \pm 0.02$  for a green broccoli and  $1.88 \pm 0.02$  for a white cauliflower, and both are independent of their directions. From the results we predict that the capacity dimensions of the bulks are about 2.7 for a green broccoli and 2.8. for a white cauliflower. The vertical cross-sections of the two plants are modeled into self-similar sets of triangular and rectangular trees. We discuss the conditions of the fractal objects in the model trees.

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A green broccoli(GB) and a white cauliflower(WC) are varieties of vegetables grown for an edible immature flower panicles or head of condensed flowers and flower stems. They are some of the most broadly nutritious of all common vegetables, and, nowadays, known as nice foods for healthy diet. More interestingly, in the view of physical science the terminal clusters of the vegetables has been known as typical fractals among living organisms[1].

Grey and Kjems discussed the fractal structure of a cauliflower, and they suggested that the fractal dimension of it could be larger than that of a broccoli[2]. Later, the cross-section of a broccoli in a vertical direction has been modeled into a self-similar set of a Pythagoras tree[3]. Recently, Romera et. al. suggested a mathematical model of the horizontal cross-section of a cauliflower as a sequence of a baby Mandelbrot set[4]. It is clear that the dimensionality is the most fundamental concept of fractal analysis. Nevertheless, surprisingly, in spite of those excellent articles, the fractal dimensions of GB and WC are not known yet.

In this article, we measure the fractal dimensions of the cross-sections of GB and WC by direct scanning method, and then predict those of the bulks theoretically. Next, we create mathematical models for the cross-sections of GB and WC, and compare those with the experimental results. The condition of creating the fractals in the mathematical models is discussed, too.

The basic concept of dimensions in our model has the property as follows: Let us introduce that  $\delta$  is a measurement scale and that the measurement is  $M_{\delta}(F)$ . Then, the fractal dimension D of a set F is determined by the power law:  $M_{\delta}(F) \sim \delta^{-D}$ . If D is a constant as  $\delta \to 0$ , F has a dimension of D[5]. The term "capacity dimension" or "box-counting dimension" is most widely used, because it can be easily applied to fractal objects. It is defined as

$$D = \frac{\log N_{\delta}(F)}{-\log \delta},\tag{1}$$

where  $N_{\delta}(F)$  is the smallest number of sets of diameter at most  $\delta$  which can cover F[6, 7]. Fractal object embedded in one- or two-dimension is easy to measure relatively compared

to other higher dimensions. However, fortunately, the dimensions of the cross-sections are generally known to be related with those of the bulk. We can propose an ansaz: Let  $D_c$  be a fractal dimension of a cross-section embedded in two dimensions. Then, that of the bulk

embedded in three dimensions can be written as

$$D = \frac{3}{2N} \sum_{i=1}^{N} D_{c_i},\tag{2}$$

where i represents every possible cross-section. If the fractal dimension of the cross-section is independent of the directions, Eq. (2) is simply written as  $D = (3/2)D_c$ .

We prepared several GB and WC about 300g in mass and 15cm in diameter. We cut half of them in horizontal direction, and the other half in vertical direction. Then, we scanned the cross-sections by a scanner and read the images into black and white. As we see in Fig. 1, the two figures by the two perpendicular directions look totally different.

Next, we read the images into matrix of numbers. The numbers are the reflectivity of each pixels. The black backgrounds produces 0's, and the white images produce large numbers of the order of  $10^5$ . Let us call the non-zero number 1 for convenience. The size of the matrix,  $M_1 \times M_2$ , is about  $10^3 \times 10^3$ . Note that the size of a pixel is an order of  $100\mu m$ .

In order to measure the fractal dimension of the cross-sections, we count the non-zero numbers by the box-counting method. It becomes the smallest number of sets to cover the white images by  $100\mu m \times 100\mu m$ . In a half reduction procedure of the matrix, a  $4\times 4$  component becomes 1 if it contains any non-zero number, otherwise it becomes 0. Then, the number of 1's becomes the smallest number of sets to cover the white images by  $200\mu m \times 200\mu m$ . And so on. In the reducing steps in half, that is  $M_1/2^n \times M_2/2^n$ , n=1,2,3..., the conversion(reverse transformation) from the reduced matrices to images is plotted in Fig. 1 for the first three steps. We see that the basic structure of the cross-sections remains unchanged in the procedure.

Fig. 2 is the log-log plot for the horizontal direction (a) and the vertical direction (b). The slopes are the capacity dimensions of the cross-sections of GB and WC. Repeating these procedures for other GB and WC, we observed similar values of  $D_c$  as  $1.78 \pm 0.02$  for GB and  $1.88 \pm 0.02$  for WC. Surprisingly, the two slopes for different directions are very similar. Therefore, from Eq. (2) we predict that the capacity dimensions are about 2.7 for GB and 2.8 for WC.

The modeling of nature should be as simple as it can, and at the same time, it should contain the basic structure of the nature. Furthermore, if some physical quantities of the model can match the real systems, it will guarantee more credits to the mathematical model. We modeled the cross-sections of GB and WC in vertical direction as a triangular tree(TT) or

a rectangular tree(RT) in Fig. 3. TT has two equilateral sides and RT has three equilateral sides. These are the simplest models of a self-similar set that has a single scale factor t or r. The scale factors t of TT and r of RT have the relations with the angles  $\theta$  and  $\phi$  in Fig. 3.

$$t = \frac{1}{2\cos\theta},$$

$$r = \frac{1}{2\cos\phi + 1}.$$
(3)

Downsizing the trees by the ratio t or r, it creates two or three branches. Therefore, the fractal dimensions of TT and RT are

$$D_c^{(t)} = \frac{\log 2}{-\log t},$$

$$D_c^{(r)} = \frac{\log 3}{-\log r}.$$
(4)

The areas of the TT and RT have the following series:  $1, 2t^2, 4t^4, 8t^6, ...(2t^2)^n, ...$  and  $1, 3r^2, 9r^4, 27r^6, ...(3r^2)^n, ...$  where n = 0, 1, 2, 3, ... Because the fractal is the limit of the series, it should converge as n increases. Therefore, the conditions of convergence of the series are  $2t^2 < 1$  and  $3r^2 < 1$ . It corresponds to  $\theta < 45^\circ$  and  $\phi < 68.5^\circ$  by Eq. (3). Note that it is clear from  $D_c < 2$  in Eq. (4).

For example, if we match the TT with GB and RT with WC, we obtain the following relations to create fractals; Since the  $D_c$  of GB is 1.78, the scale factor t=0.68 and the angle  $\theta=43^{\circ}$  from Eqs. (3) and (4). By the same method, if the  $D_c$  of the WC is 1.88, then r=0.56 and  $\phi=67^{\circ}$ . As the angle  $\theta$  approaches 45° or  $\phi$  approaches 68.5°, the dimensions of the trees go to two because the limit bents to cover the two dimensional surface. On the other hand, as the angle  $\theta$  or  $\phi$  approaches 0°, the dimension of the tree goes to one because the limit goes to a long rod.

We measured the fractal dimensions of the cross-sections of a green broccoli and white cauliflower by a direct scanning method. They were  $1.78 \pm 0.02$  and  $1.88 \pm 0.02$ , and almost independent of the directions of the cross-sections. From these results we predict that the fractal dimensions of the bulks are about 2.7 for GB and 2.8 for WC.

We created mathematical models for the vertical directions with only one scale factor. It is a triangular or a rectangular tree of three or four equilateral sides. We suggested the condition of creating fractals in our model, and matched the models into experimental measurements. This method of the scanning cross-sections and mathematical models from polygonal trees can be widely applied to many complex bulk fractals in nature.

- FIG. 1: The scanned image, top left, and the images from half-reduction procedure and reverse transformation. From the top left and clockwise direction,  $M_1 \times M_2$ ,  $M_1/2 \times M_2/2$ ,  $M_1/4 \times M_2/4$ , and  $M_1/8 \times M_2/8$ . (a) Horizontal images of a green broccoli. (b) Vertical images of a green broccoli. (c) Horizontal images of a white cauliflower.
- FIG. 2: The log-log plots of the two perpendicular directions. The slopes are  $D'_cs$  (a) a green broccoli, (b) a white cauliflower
- FIG. 3: The self-similar sets with single scale factors each. (a) Triangular tree(TT), (b) Rectangular tree(RT).

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